MY CHECKING SYSTEM - and how to use it

by George Lane, presented at the Mental Calculation World Cup 2014 in Dresden

This checking system is based mainly on modular calculations, along with a few other very simple checks which can help to locate where any necessary correction may need to be made.

Let us first look at a simple test for modularity with regard to the number nine. This is so simple that many schoolchildren do it as a game, without fully realising its significance. By adding the values of the digits of a number together, a total is easily reached. If this total has more than one figure, add these together too - and keep going until you reach a single figure. For example, we can look at the number 385,499 and say that 3+8+5+4+9+9=38, then 3+8=11, and finally 1+1=2 - which is the answer; that is to say that the number 385,499 is 2 modulo 9.. This single figure result is what I call the 'marker' of the original number. This process is also known as 'casting out nines', since that is exactly what it does - it removes the greatest available multiple of nine, with the possible exception of leaving the number nine as the marker.

Another very popular method of finding such a modular marker of this kind, this one being relative to the number eleven, is performed by alternately subtracting and adding the digits of a number, starting from the right hand end, and beginning with a subtraction. By adding or subtracting multiples of 11 in order to keep the total positive but lower than 11, the final result will be the modularity of the number with regard to 11. Using the same number as before, i.e. 385,499, we have 9-9+4-5+8-3=4, and so we see that the number 385,499 is 4 modulo 11.

Modular calculations are well suited to prime number bases. With that in mind, we can now consider the fact that three consecutive prime numbers, specifically 7, 11 and 13, multiply out to give 1,001. Just as we have already seen that we can remove multiples of 11 by alternately subtracting and adding the single digits of a number, we can also remove multiples of 1,001 by subtracting and adding blocks of three digits at a time. This has the consequent advantage of removing multiples of these three prime numbers all at once, and leaving a number with a value no greater than 1,000 to examine. Since the modular markers of numbers behave in the same manner as the numbers they represent, this advantage in indeed considerable. Using the same number yet again, we have 499-385=114, and so the number 114 will have the same modular markers with regard to 7, 11, and 13 as the original number 385,499. These values are 2 modulo 7, 4 modulo 11, and 10 modulo 13.

Now let's consider how we can use this information to check the answer to a multiplication problem, specifically that of 385,499 x 17,357,561. When I performed this multiplication myself, as part of the testing of my training system, I arrived at the answer 6,691,522,407,939.

Using the same process to find what I call the 'K1 register' of the other number, this one being 17,357,561, we have 561-357=204, and 204+17=221. This number is 4 modulo 7, 1 modulo 11, and 0 modulo 13.

By multiplying out the modular markers of the two numbers, we can find those of the correct solution. 2x4=8, which is 1 modulo 7, then we see 4x1=4, which is - of course - 4 modulo 11, and finally we have 10x0=0 which is quite obviously 0 modulo 13. These are therefore the modular markers we should get from the answer, if it is correct. Looking at the answer, we have 939-407+522-691+6=369. This number is 5 modulo 7, 6 modulo 11, and 5 modulo 13. The answer is wrong, but this need not be a total disaster. We know what the modular markers of the answer should be, so we can work out the K1 register also.

Starting with the fact that the correct answer is 1 modulo 7, we take the 1 as a starting number and add a sufficient multiple of 7 to create a number which is 4 modulo 11 - as is also required by a correct answer. We could add a single seven at a time and check each of the results until we reach the required value, but there is a short cut. This makes use of the fact that 3x7=21, which is one single point short of being an exact multiple of 11. If we consider that the number 1 is clearly 1 modulo 11, and we need to reach a number which is 4 modulo 11, we can step down one unit at a time from 1 until we reach 4. But since our target value of 4 is actually greater than our starting value of 1, we must move in the opposite direction instead. As we are seeking modularity with regard to the number 11, we must subtract the number of upward steps (i.e. 3) from 11 to find the equivalent number of downward steps - which is 8. We triple this number, giving us 24, and subtract the biggest multiple of 11 available (clearly 2x11=22) to give a value of 2. This is the number of sevens we must add to the starting value of 1 in order to reach the first available number which is also 4 modulo 11. 2x7=14, and 1+14=15.

Now we need to raise this number by a sufficient multiple of 77, maintaining the two modular markers we have already identified, to reach a number which is 0 modulo 13. There is a short cut here too, and it is similar to the previous one - and it is simpler as well. We can use the same kind of logic as before, but we don't need to triple anything as the number 77 is already one unit short of being an exact multiple of 13. Our present number 15 is 2 modulo 13, and the target value is 0 modulo 13. Since we reach this target by stepping down just twice from 2 modulo 13, we see that we need to add 2x77 (=154) to our present number of 15, giving us a final result of 169. This is therefore the K1 register of the correct solution.

Looking back at the K1 register of the original - and wrong - answer, we can see that the original register is exactly 200 too great. Now considering the markers of the numbers, we have already found that the marker of 385,499 is 2. The number 17,357,561 has a marker of 8, and 2x8=16 which has a marker of 7. The marker of the original answer is 9, which is 2 too high. Bearing these two differences in mind, the indication is that the first figure of one of the three-figure blocks can be reduced in value by 2 in order to rectify both problems. Since both the marker and the K1 register need to be reduced, the block concerned must be one of those which is added to the total when calculating the K1 register. If one had value needed to be reduced and the other increased, the relevant block would have been one of those which is subtracted.

We now know that the error is in either the final three-figure block or the block which is two places to the left of this one. There are indeed essentially five such blocks in the answer, but the first - which would otherwise also need to be considered - is too short, with only one figure. To reduce this value by 200 would take it into the negative range, which is ridiculous. A quick check of the last three figures of the answer shows that they are all correct, and so the error must therefore be in the earlier block. Reducing the value of that block from 522 to 322 leaves us with an amended answer of 6,691,322,507,939 - which is correct.

Can this system be used in reverse to help create answers rather than just checking them? Yes, it can. Let's look at the multiplication problem 58,279 x 29,738. The K1 registers of these numbers are 221 and 709 respectively. Moving more quickly now, and taking the modular markers of these numbers, we can find the K1 register of the solution to this problem. 4x2=8 which is 1 modulo 7, 1x5=5 which is 5 modulo 11, and 0 multiplied by anything is always 0, which is 0 modulo anything.. Starting with 1, i.e. the lowest number to be 1 modulo 7, we add 10x7=70 to get 71, which is 6 modulo 13. Now stepping down six times to reach a value of 0 modulo 13, we add 6x77=462 to 71 gives us 533, which is therefore the K1 register of the correct answer.

A very quick estimation shows us that the answer has ten figures and starts with 173. Another quick check, looking at the last three figures of each number, shows that the last three figures of the answer are 902. Using all this information, we can see that the two three-digit blocks to be added when calculating the K1 register are 902 and something over 730. The sub-total of these two blocks is therefore in the range of 1,632 to 1,641. As the first of the four blocks in the answer is just the initial 1, and this is to be subtracted, this subtotal is now in the range of 1,631 to 1,640.

Since the K1 register of the solution is 533, and this system operates on multiples of 1,001, we can add 1,001 to 533 to give us the value of 1,534 which we can now compare with the range we have just identified. Consequently, we can see that the penultimate three-figure block must be in the range of 097 to 106, depending on the nature of the fourth figure of the correct solution.

Now we can look at the markers of the numbers involved. The marker of 58,279 is 4, and that of 29,738 is 2. 4x2=8, and so the marker of the final answer must also be 8. If we assume that the fourth figure of the answer is zero, with value of the penultimate three-figure block therefore being 097, the marker of the provisional answer 1,730,097,902 is 2. This is six points too low. For each single unit of increase to the fourth figure, there must also be a similar increase to the penultimate block in order to maintain the K1 register. The marker of the provisional answer will thus be raised by two points at a time. Since three such increases give us the marker of 8 which we require, we add a value of 3 to each of these numbers. The block following the initial 1 thus becomes 733, the next block becomes 100, and the final block is already known to be 902. The final answer is therefore 1,733,100,902, which is correct.

A point on which to conclude: When considering the difference between the required marker and the provisional one, the difference may be odd - or possibly negative - but this is very easy to overcome. Simply add 9 or 18 to the target marker in order to create a positive and even difference. And, of course, the provisional solution is already correct if the difference is zero. The only exception to this is the fact that if there is a possibility of each if the ‘missing’ figures being zero simultaneously, with a difference between the actual and required markers also being zero, then there is also the opportunity for both of the missing figures to be nines. This, however, can easily be resolved by way of a quick visual inspection – either the zero option would be too low or the nine option would be too high.